EX 30 :

1) $I+J=\int_0^{\frac{\pi}{2}} 2^x(\cos^2 x+\sin^2 x)dx=\int_0^{\frac{\pi}{2}} 2^x dx$

$=\left[\frac{1}{\ln 2}2^x\right]_0^{\frac{\pi}{2}}=\frac{2^{\frac{\pi}{2}}-1}{\ln 2}$

$I+J=\frac{\sqrt{2^\pi}-1}{\ln 2}$

2) $K=\int_0^{\frac{\pi}{2}} 2^x.cox(2x)dx$

$\begin{cases} U'(x)=\cos 2x \to U(x)=\frac{1}{2}\sin(2x) \\ V(x)=2^x \to V'(x)=2^x.\ln 2 \end{cases}$

$K=\left[\frac{2^x\sin(2x)}{2}\right]_0^{\frac{\pi}{2}}-\frac{\ln 2}{2}\int_0^{\frac{\pi}{2}} 2^x\sin(2x)dx$

$K=\frac{-\ln 2}{2}\int_0^{\frac{\pi}{2}} 2^x.\sin(2x)dx$

$\begin{cases} U'(x)=\sin 2x \to U(x)=\frac{-1}{2}\cos 2x \\ V(x)=2^x \to V'(x)=2^x.\ln 2 \end{cases}$

$K=\frac{-\ln 2}{2}\left(\left[\frac{-2^x.\cos 2x}{2}\right]_0^{\frac{\pi}{2}}+\frac{\ln 2}{2}\int_0^{\frac{\pi}{2}} 2^x\cos 2x\right)$

$K=\frac{-\ln 2}{2}\left(\frac{1+\sqrt{2^\pi}}{2}+\frac{\ln 2}{2}.K\right)$

$*K+\left(\frac{\ln 2}{2}\right)^2.K=\frac{-\ln 2}{2}\left(\frac{1+\sqrt{2^\pi}}{2}\right)$

$K=\frac{-\ln 2}{4+(\ln 2)^2}(1+\sqrt{2^\pi})$

$*I-J=\int_0^{\frac{\pi}{2}} 2^x(\cos^2 x-\sin^2 x)dx=\int_0^{\frac{\pi}{2}} 2^x\cos(2x)dx=K$

d'où $I-J=\frac{-\ln 2}{4+(\ln 2)^2}(1+\sqrt{2^\pi})$

3) $\begin{cases} I+J=\frac{\sqrt{2^\pi}-1}{\ln 2} \\ I-J=\frac{-\ln 2}{4+(\ln 2)^2}(1+\sqrt{2^\pi}) \end{cases}$

Qarray.tn -Aqra Blech- Qarray.tn